FINITE ELEMENT MODELS OF WIRE ROPE

FOR VIBRATION ANALYSIS

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Abstract

The usefulness of wire rope in shock and vibration isolation is briefly reviewed and its modeling for the purpose of vibration analysis is addressed. A model of a nominally straight segment of wire rope is described in which the rope structure is represented by a maiden, or central, strand of wire with one (or more) strand(s) wrapped around it in a helix (helices). The individual strands are modeled using finite elements and MSC NASTRAN. Small linear segments of each wire are modeled mathematically by dividing them lengthwise into triangular prisms (thick "pieces of pie") representing each prism by a solid NASTRAN element. To model pretensioning and allow for extraction of internal force information from the NASTRAN model, the "wound" strands are connected to the maiden strand and each other using "spring" (scalar elastic) elements. Mode shapes for a length of wire rope with one end fixed to a moving base and the other attached to a "point" mass, are presented. The use of the NASTRAN derived mode shapes to approximate internal normal forces in equations of motion for vibration analyses is considered.

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Introduction

Wire rope[1] is, from the basic point of view, simply several strands of wire twisted, or wound, together (see Fig. 1). Some types are commonly called "cable" and are used to carry electricity, support bridges and "cable cars," raise and lower heavy loads and in many other practical ways. A less obvious, but equally important, use of wire rope is in shock and vibration isolation devices.[2,3] The structure of wire rope provides many interfaces at which a portion of the relative motion of strands of wire is converted by friction into heat, thereby dissipating vibrational energy.[4] Furthermore, the stiffness of wire rope structures can be tailored to provide support and restoring forces. Stiffness and damping are adjusted by varying wire diameter, the number of strands, pretensioning and the arrangement of lengths of the wire rope. Commonly, helical coils of ropes[2,3] are fixed in clamps (see Fig. 2) to form individual shock and/or vibration isolators. The isolators are used to support and isolate communications equipment in vehicles which are subjected to large magnitude, short-term accelerations; i.e., "shocks." In addition to absorbing shock, the internal, or system, damping,[4,5] of the wire rope devices provides vibrational isolation over wide ranges of frequencies and amplitudes.

The damping characteristics of wire rope and vibration isolators made from it are not well understood from the theoretical standpoint. Apparently, the design of individual isolators is accomplished by experimentation by engineers with considerable experience in applications of these devices.[2] Realistic mathematical models of wire rope isolators would be useful in the design process and perhaps would allow the achievement of the confidence levels in isolator characteristics needed for more applications in which
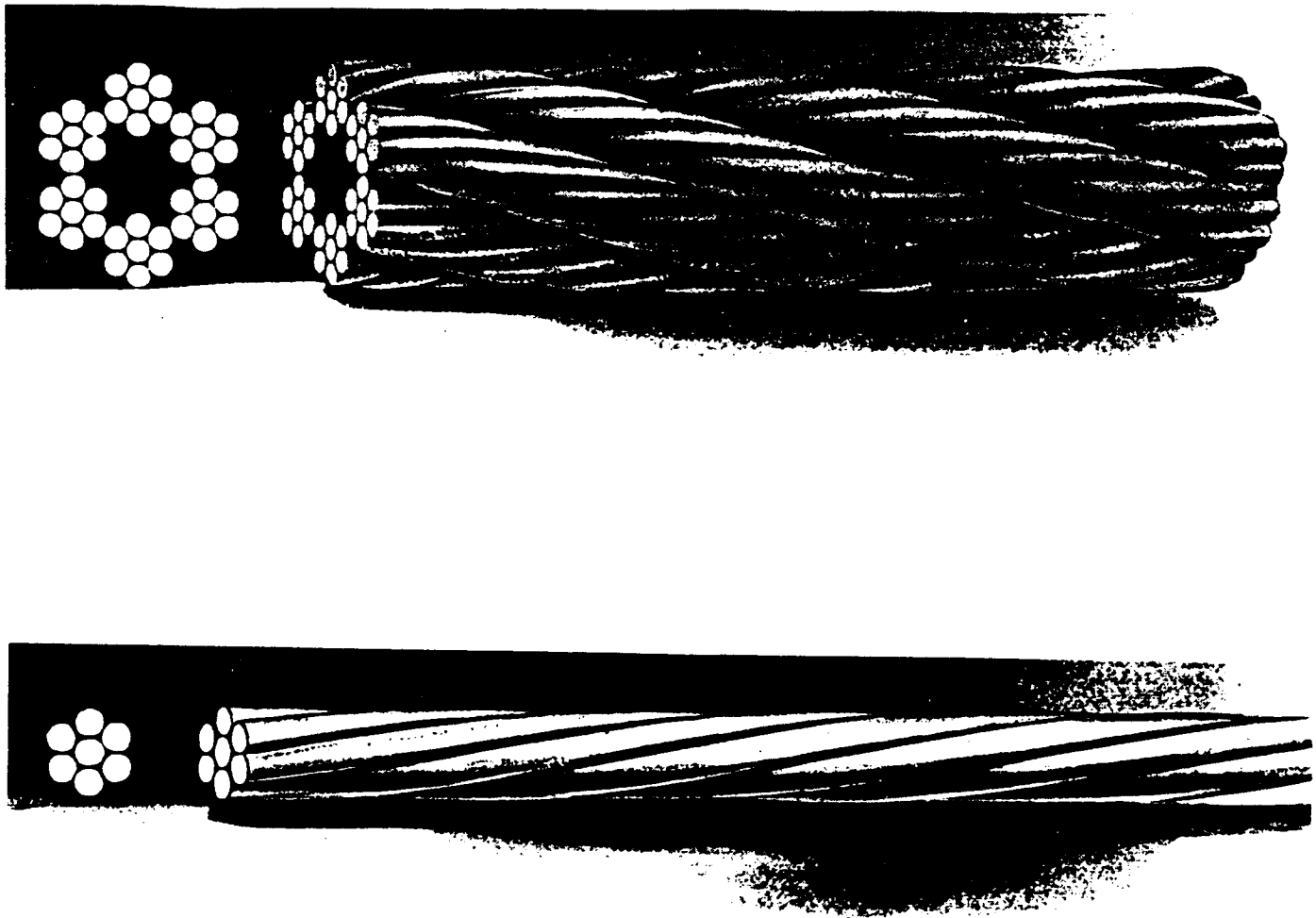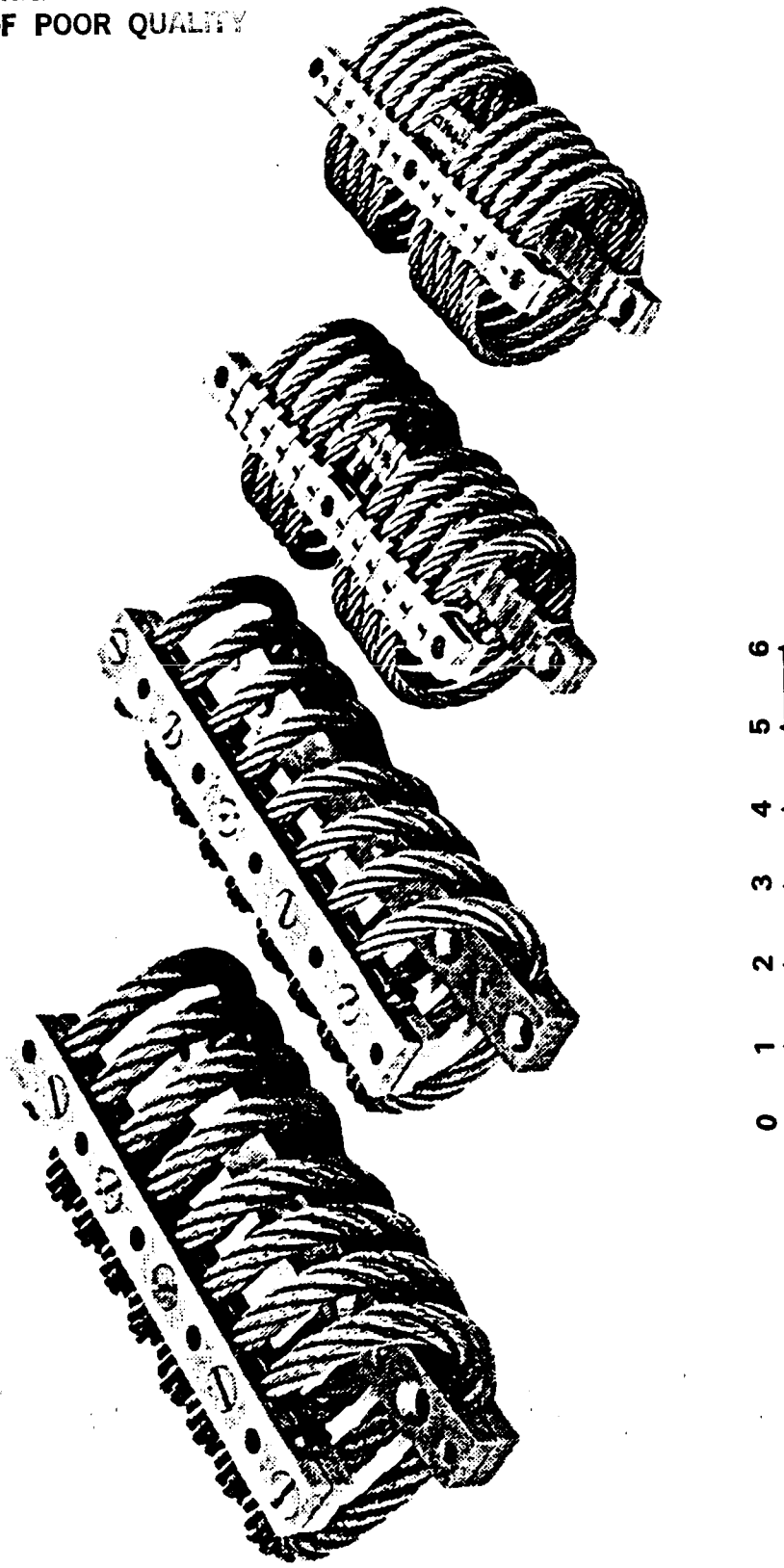
Fig. 1. Wire rope.

Fig. 2. Helical isolators.

damping rates and dynamic response must be very accurately known to prevent resonance and control interaction problems.

The purposes of this paper are (1) to present the results of work on the problem of obtaining accurate mathematical models of wire rope isolators using MSC NASTRAN and (2) to present some preliminary results of work toward obtaining equations of motion of a wire rope "pendulum" for use in correlating experimental and theoretical results.

Wire Rope Model

Basic Model

A sketch of a "wire rope" segment of length L, composed of a maiden wire strand and a single "wound" strand, is shown in Fig. 3. It is assumed that when the rope is in its undeformed state the angle of the helix formed by the centerline of the wound strand, α, is constant over the length L. For the finite element analysis, the maiden strand is divided into 372 elements such that each cross section of the strand looks like a hexagon cut into triangular pieces which have thicknesses equal to the radius of the strand. The other strands are modeled in a similar manner. In addition, scalar elastic elements ("springs") are used to connect the wound and maiden strands. These springs are incorporated so that the distribution of the normal force between the two strands can be determined for each mode shape computed (see Fig. 5). NASTRAN produced figures showing two-strand and seven-strand wire rope are given as Figures 6 and 7, respectively.

Boundary Conditions

The boundary conditions are chosen so that the "top" of the segment is fixed. Two choices for the boundary conditions on the other end are considered here. The first is to use a "free" end condition, (Case 1). The
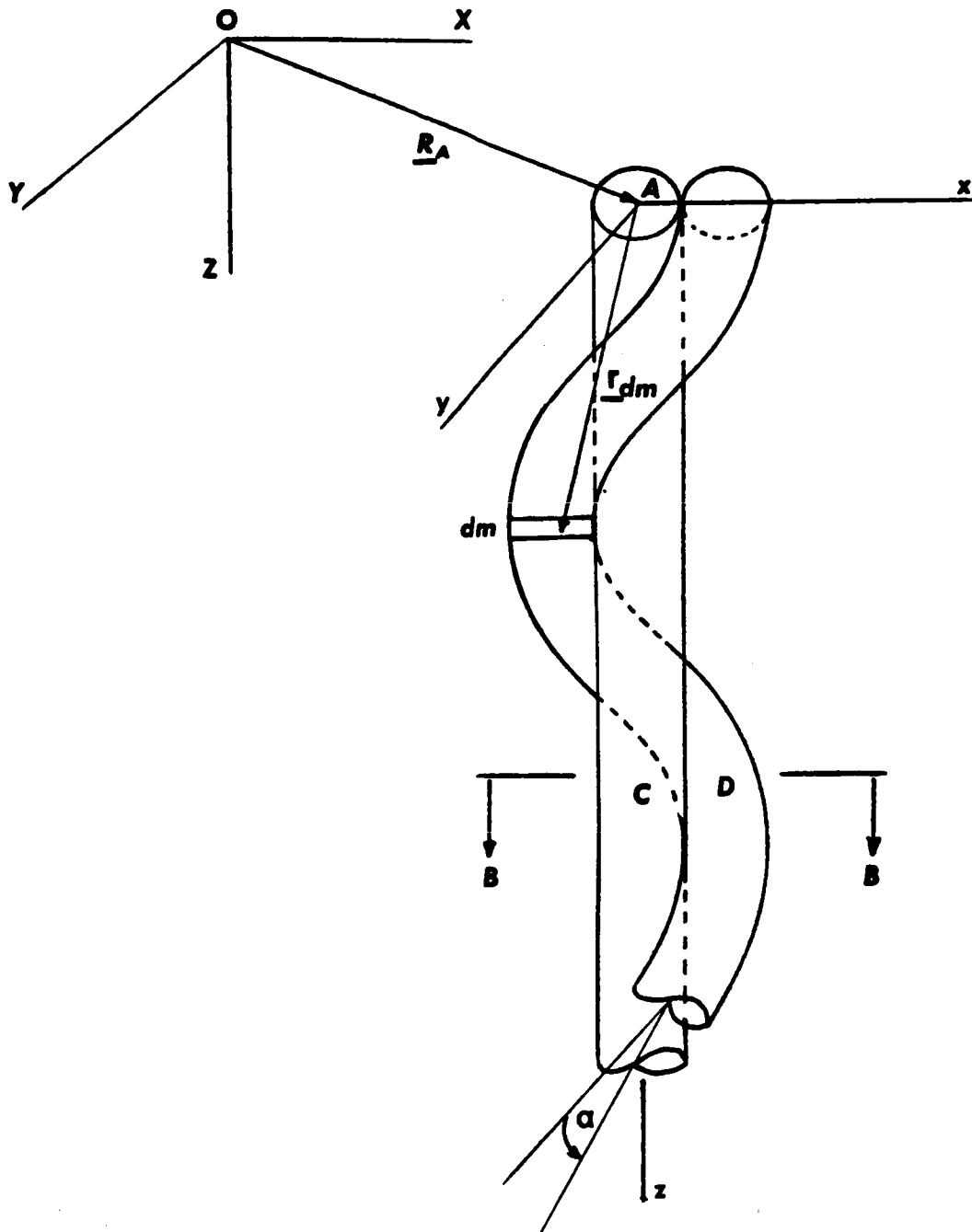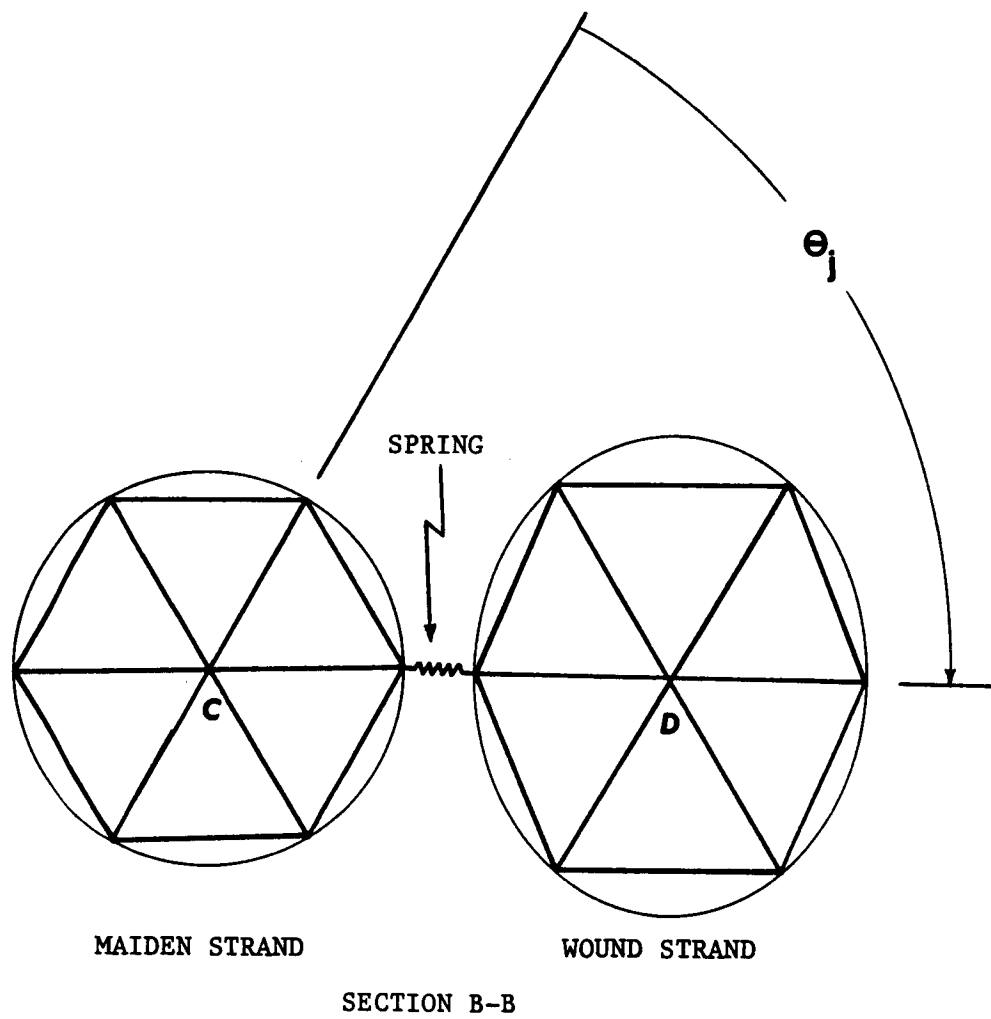
Fig. 3. Two-strand wire rope.

SPRING

Θ_j

MAIDEN STRAND   WOUND STRAND

SECTION B-B

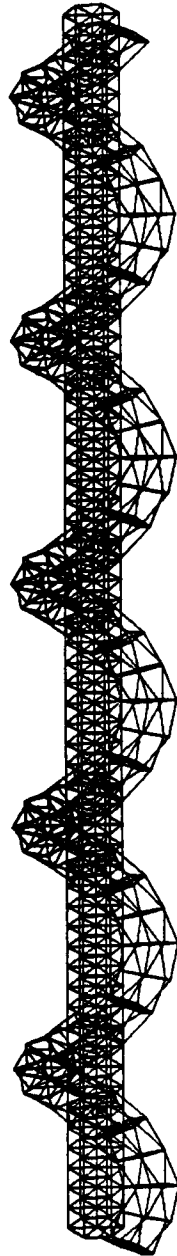Fig. 4. Cross section of two-strand wire rope showing finite elements.
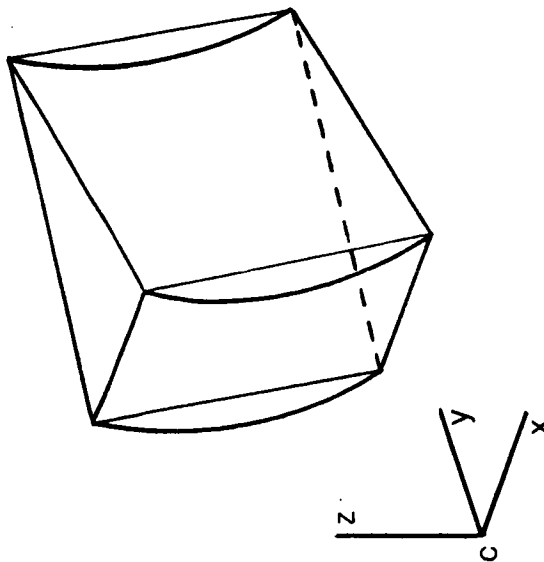
Fig. 5. PENTA MSC/NASTRAN solid element.

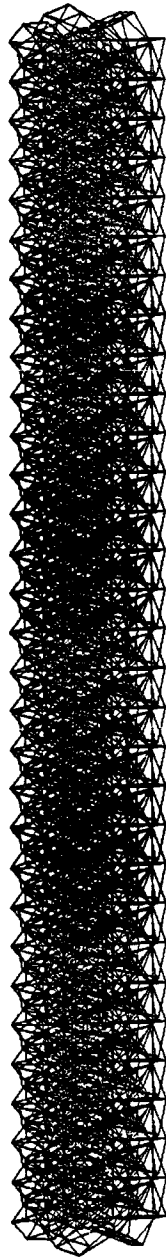Fig. 6.   NASTRAN model of two-strand wire rope.

Fig. 7.   NASTRAN model of seven-strand wire rope.

second choice (Case 2) is to attach to the "bottom" end a mass which is relatively large compared to the mass of the rope segment.

The fourth mode shape for the seven-strand rope alone (Case 1) is shown in Fig. 8. The corresponding mode shape for the Case 2 "pendulum" model is given in Fig. 9.

Equations of Motion

Our stated objective is to obtain "analytical" models of wire rope vibration isolation. Here, "analytical" implies that mode shapes from NASTRAN models without damping will be used rather than analytical mode shapes such as the classical ones for beams. The mode shapes are to be used to approximate the displacements of the wire rope and attach "rigid" bodies when linear damping is present and when nonlinear nonconservative and conservative forces act on the system. The wire rope "pendulum" is a simple system which we hope to use to test this method for modeling these effects.

Except for the determination of nonconservative internal forces, the most attractive way of deriving equations of motion for the pendulum is to use the Lagrangian formulation.

The Lagrangian

To obtain the Lagrangian $\mathcal{L}$, we must write the kinetic and potential energies of the pendulum. The support point, A, is assumed to move in a prescribed manner so that the vector $\underline{R}_A$ from the fixed point 0 is known. The kinetic energy, T, of the pendulum, assuming the mass M is a "point mass," is

$$T = \frac{1}{2} \sum_{i=1}^{N} \int_{m_i} (\underline{V}_A + \underline{\dot{r}}_{dm}) \cdot (\underline{V}_A + \underline{\dot{r}}_{dm}) dm$$

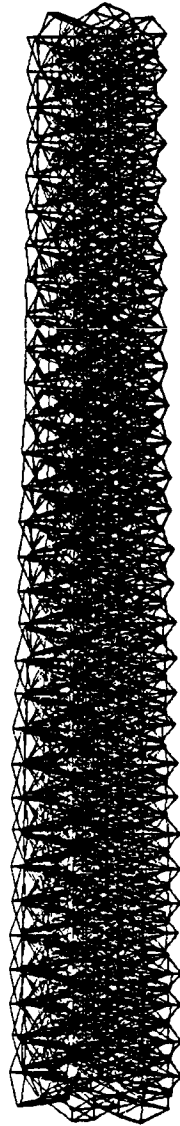$$+ 1/2 \ M \ (\underline{V}_A + \underline{\dot{r}}_M) \cdot (\underline{V}_A + \underline{\dot{r}}_M) \tag{1}$$

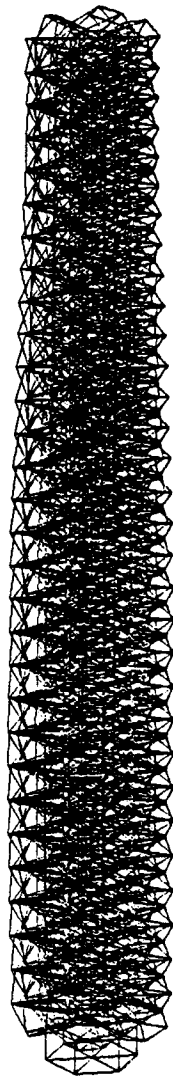Fig. 8 Mode shape for wire rope model, Case 1.

C-7

Fig. 9 Mode shape for wire rope model, Case 2.

where $\underline{V}_A = \dot{\underline{R}}_A$, the velocity of A, $\dot{\underline{r}}_{dm}$ is the velocity of a generic mass element dm, N is the number of strands (see Fig. 3), and m_i is the mass of the ith strand. We approximate $\underline{r}_{dm}$ by $\sum\limits_{j=1}^{m} \underline{\phi}_j \, q_j$, where the $\underline{\phi}_j$ are mode shapes, the q_j are generalized coordinates, m is the number of modes used and $\underline{r}_M = \sum\limits_{j=1}^{m} \underline{\phi}_j \, (0 \ 0 \ L)q_j$. The $\underline{\phi}_j$ are functions of the coordinates x_o, y_o and z_o of dm in the undeformed "structure" and the q_j are, of course, functions of time to be determined.

In matrix form, Eq. (1) is

$$T = \frac{1}{2} M_T \, \underline{V}_A^{\,2} + M \, \underline{V}_A^T \, \underline{\phi}_L \, \dot{\underline{q}} + \frac{1}{2} \dot{\underline{q}}^T \underline{\phi}_L^T \underline{\phi}_L \dot{\underline{q}} M$$

$$+ \underline{V}_A^T \sum_{i=1}^{N} \int_{m_i} \underline{\phi} \, \dot{\underline{q}} \, dm + \sum_{i=1}^{N} \frac{1}{2} \int_{m_i} \dot{\underline{q}}^T \underline{\phi}^T \underline{\phi} \, \dot{\underline{q}} \, dm \qquad (2)$$

where M_T is the total mass of the system, $\underline{q} = (q_1 q_2 \cdots q_m)^T$ and $\underline{\phi} = (\underline{\phi}_1 \ \underline{\phi}_2 \cdots \underline{\phi}_m)$ and $\underline{\phi}_L$ is $\underline{\phi}$ evaluated at $x_o = y_o = 0$, $z_o = L$.

As expected, we may write

$$T = \frac{1}{2} \dot{\underline{q}}^T \underline{\underline{M}} \, \dot{\underline{q}} + \underline{a}^T \, \dot{\underline{q}} + \underline{b} \quad , \qquad (3)$$

where

$$\underline{\underline{M}} = \sum_{i=1}^{N} \int_{m_i} \underline{\phi}^T \underline{\phi} \, dm + M \, \underline{\phi}_L^T \, \underline{\phi}_L \qquad (4a)$$

$$\underline{a}^T = M \underline{V}_A^T \underline{\phi}_L + \underline{V}_A^T \sum_{i=1}^{N} \int_{m_i} \underline{\phi} \, dm \qquad (4b)$$

and

$$\underline{b} = \frac{1}{2} M_T \underline{V}_A^{\,2} \qquad (4c)$$

the potential energy, V, is due to the elastic stiffness of the system and to gravity. For the former, write

$$V_s = 1/2 \; q^T \; \underline{\underline{K}}_s q \tag{5}$$

where $\underline{\underline{K}}_s$ is the stiffness matrix.

The potential energy due to the location elements of mass in the rope and the mass M in a uniform gravity field is obtained by assuming that M is a point mass at the end of the length of rope and that the rope has mass per unit length $\sigma(z_0)$. Furthermore, the motion is restricted to the xz-plane and $dx/dz_0 = (100) \; (d\underline{\phi}/dz_0 \; q)$ is assumed small enough for terms of order q_j^3 and higher to be neglected. Then, the potential energy contribution due to gravity can first be written as

$$V_g = \int_0^L Mg(1 - \cos \theta) dz_0$$

$$+ \int_0^L \sigma[1 - \int_0^{z_0} \cos[\theta(\zeta)] d\zeta] dz_0 \tag{6}$$

By noting that $\cos\theta \simeq 1 - \frac{1}{2} (dx/dz_0)^2$, we may write

$$V_g = \frac{1}{2} M \int_0^L (\frac{dx}{dz_0})^2 \; dz_0$$

$$+ \frac{1}{2} \int_0^L \sigma \int_0^{z_0} [\frac{dx}{dz_0} \Big|_{z_0 = \zeta}]^2 \; d\zeta \; dz_0 \tag{7}$$

Obviously, V_g is of the form,

$$V_g = \frac{1}{2} q^T \; \underline{\underline{K}}_g \; q \tag{10}$$

The nonconservative forces acting on dm are assumed to be due to viscosity and coulomb friction between the strands. Viscous forces can be included in the equations by using some form of modal damping. For the coulomb friction, we need to consider the rope's structure.

Let the strands be renumbered so that the maiden strand is i=0, and at any value of z_0, the other strands are numbered 1 through N-1 in a counterclockwise manner about the z_0 axis. Also, let $\underline{f}_{dm_i}$ denote the force on denote the force on the element dm_i of the ith strand and let μ denote the coefficient of friction. Furthermore, let f_{ij} denote the magnitude of the normal force between the ith and the jth strands. Then, for dm_0, we have the coulomb friction force,

$$\underline{f}_{dm_o} = \sum_{i=1}^{N-1} \mu \, f_{oi} \, \text{sgn}[(\underline{\dot{r}}_{dm_o} - \underline{\dot{r}}_{dm_i}) \cdot (\hat{\underline{t}}_i \hat{\underline{t}}_i + \hat{\underline{b}}_i \hat{\underline{b}}_i)] \qquad (11)$$

where $\hat{t}_i$ and $\hat{b}_i$ are the tangent and bi-normal unit vectors, respectively, of dm_i. For dm_j, $j > 0$, we may write.

$$\underline{f}_{dm_j} = + \mu f_{oj} \, \text{sgn}[(\underline{\dot{r}}_{dm_o} - \dot{r}_{dm_j}) \cdot (\hat{\underline{t}}_j \hat{\underline{t}}_j + \hat{\underline{b}}_j \hat{\underline{b}}_j)$$

$$- \mu f_{jk} \, \text{sgn}[(\underline{\dot{r}}_{dm_j} - \underline{\dot{r}}_{dm_k}) \cdot (\hat{\underline{t}}_j \hat{\underline{t}}_j + \hat{\underline{b}}_j \hat{\underline{b}}_j)$$

$$- \mu \, f_{j\ell} \, \text{sgn}[(\underline{\dot{r}}_{dm_j} - \underline{\dot{r}}_{dm_\ell}) \cdot (\hat{\underline{t}}_j \hat{\underline{t}}_j + \hat{\underline{b}}_j \hat{\underline{b}}_j) \qquad (12)$$

with j, k and ℓ in the following triplets:

$$\begin{bmatrix} j \\ k \\ \ell \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix}, \dots, \begin{bmatrix} N-1 \\ 1 \\ N-2 \end{bmatrix} \qquad (13)$$

In Eqs. (12) we have also used[6]

$$\hat{t}_j = \cos\alpha\,[-\sin\theta_j\,\hat{i} + \cos\theta_j\hat{j}] + \sin\alpha\,\hat{k} \tag{14a}$$

and

$$\hat{b}_j = \sin\alpha[\sin\theta_j\,\hat{i} - \cos\theta_j\hat{j}] + \cos\alpha\,\hat{k} \tag{14b}$$

where $\theta_j(z_o)$ is the angle of the centerline of the jth strand at z_o.

Because $\underline{r}_{dm_i} = \underline{R}_A + \underline{\Phi}(x_o,y_o,z_o)\,\underline{q}$, where (x_o,y_o,z_o) is a point in the undeformed i^{th} strand, $\delta\underline{r}_{dm_i} = \underline{\Phi}(x_o,y_o,z_o)\delta\underline{q}$. The generalized force matrix is therefore[7]

$$\underline{Q} = \sum_{i=0}^{N-1} \int_{m_i} \underline{\Phi}^T \underline{f}_{dm_i}, \tag{15}$$

where the integral indicates a summation of the quantities within it.

The equations of motion then follow from $\mathcal{L} = T - V_s - V_g$, viz.,

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\underline{q}}} - \frac{\partial\mathcal{L}}{\partial\underline{q}} = \underline{Q}^T - (\underline{\underline{D}}\,\dot{\underline{q}})^T, \tag{16}$$

where $-\underline{\underline{D}}\,\dot{\underline{q}}$ is the generalized viscous damping force. For normal modes of the undamped structure, we have (with $\underline{\underline{D}}$ a diagonal matrix),

$$M_j\ddot{q}_j + k_j q_j + \underline{K}_{gj}^T\underline{q} = Q_j - d_j\dot{q}_j - \dot{a}_j \tag{17}$$

where M_j is the generalized mass and k_j the generalized stiffness, respectively, for the jth mode $\underline{K}_{gj}^T$ is the jth row of $\underline{\underline{K}}_g$, Q_j is the jth element of $\underline{Q}$, d_j the diagonal element of $\underline{\underline{D}}$ in the jth row and a_j is the jth element of $\underline{a}$.

The motion in q_j is coupled in two ways. First, through the gravity terms and, second, through the Q_j since up to m of the $\dot{q}_k$ appear in Q_j to determine its sign.

The next steps to be taken are (1) to obtain experimental data for the position of M and (2) to match it with

$$\underline{R}_M = \underline{R}_A + \underline{r}_M \tag{18}$$

which can be determined when $\underline{R}_A$ is specified and the q_i are known.

Summary

The use of wire rope in shock and vibration isolation devices has been reviewed briefly. Finite element models of a nominally straight length of wire rope have been described. These models were developed as part of an effort to construct mathematical models of wire rope for use in the analysis of vibration isolation devices which are constructed from wire rope.

Equations of motion for one of the finite element models, a seven-strand rope suspended from one end and with a mass attached to the other end to form a "pendulum," were derived. The use of these equations to simulate the motion of the pendulum was discussed.

Further work needs to be done with the current models and new models of helical isolators should be developed.

Acknowledgment

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